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Market Fundamentals Versus Speculative Bubbles: The Case of Stock Prices in the United States

Louis O. Scott

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The Case of Stock Prices in the United States

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ABSTRACT

The possibility that stock prices deviate substantially from market fundamentals and are characterized by speculative bubbles is explored. A regression test is constructed by using a nonseparable intertemporal utility function with a consumption-based asset pricing model. The regression test is applied to a portfolio of all stocks on the NYSE as well as individual stocks, and we find evidence that stock prices have deviated substantially from market fundamentals for the post World War II period. These results are then contrasted with tests that generally support the efficient markets hypothesis.

MARKET FUNDAMENTALS VERSUS SPECULATIVE BUBBLES: THE CASE OF STOCK PRICES IN THE UNITED STATES

Economists and observers of financial markets have been fascinated with pathological price movements in financial markets for many years. Early historical episodes include the tulip bulb craze in Holland, the Mississippi bubble in France, and the South Sea bubble in Britain.[1] In the early part of this century in the United States, we have the Florida real estate craze of the 1920's and the dramatic rise in stock prices from 1928 to 1929, which resulted in the great stock market crash. Some of these episodes involve fraud, and one might argue that bubbles and pathological price changes do not occur in our sophisticated modern markets. Malkiel (1985, Ch. 3), however, describes some of the more recent speculative crazes of the U.S. stock market such as the fascination with growth stocks, new issues, and conglomerates during the 1960's and the rise of high technology stocks in 1983. And of course, we have real estate prices in southern California during the 1970's. It would appear that our modern financial markets are not immune to dramatic price changes which are based on something other than market fundamentals. These bizarre price changes are sometimes called speculative bubbles, and professional interest in the subject has increased in recent years because bubbles frequently appear in rational expectations models. Recent discussions on the relevance of bubbles are included in the survey on rational expectations and macroeconomic policy by Turnovsky (1984) and in the survey of recent work on business cycles by Zarnowitz (1985). An important feature of Keynes' General Theory is the "animal spirits" theory of investment, which attributes similar behavior to businessmen in making capital investment decisions.

In recent years, a theoretical literature has developed which attempts to determine if speculative bubbles (or pathological price changes) can arise in asset markets with rational optimizing behavior by individuals.[2] Kass and Shell (1983) find that bubbles or extrinsic uncertainty can play a role in an overlapping generations model. Tirole (1982) finds that bubbles can arise in a myopic rational expectations equilibrium, but not in a fully dynamic rational expectations equilibrium. In some models, there is either a terminal or transversality condition which can rule out arbitrary price solutions or bubbles. An asset which has a finite life, for example, has a natural terminal condition (maturity) which rules out the possibility of a bubble, so that securities like government bonds and corporate bonds should not experience speculative bubbles.[3] The possibility remains for long-lived securities or securities with infinite lives such as common stocks and land. In some models, such as the asset pricing model of Lucas (1978), there is a transversality condition which arises from the dynamic optimizing behavior of agents which will rule out the possibility of speculative bubbles for infinitely-lived assets. These technical points are explored in more detail in the next section where we examine the possibility of bubbles in a simple asset pricing model. In summary, the theoretical results on speculative bubbles are mixed and it is clear that short-run optimizing behavior and rational expectations are not sufficient for ruling out speculative bubbles. The empirical issue examined in this paper is whether speculative bubbles play a significant role in financial markets.

Constructing empirical tests for bubbles is not easy. A price bubble to one observer might be a large forecast error in market fundamentals to another observer; any test of price bubbles will require a model of market fundamentals. Flood and Garber (1980) present the first tests for bubbles in a rational expectations model by testing for bubbles in the general price level during the German hyperinflation of the 1920's; their model is Cagan's money demand equation. They found no evidence of bubbles, but subsequent work by Burmeister and Wall (1982) and Flood, Garber, and Scott (1984) did find evidence of bubbles. Formal tests for bubbles in the stock market are rare but Blanchard and Watson (1982), for example, have interpreted the results of Shiller (1981a,b) and LeRoy and Porter (1981) on variance bounds tests for stock prices as evidence of a stochastic bubble in the stock market.[4] Shiller and LeRoy and Porter use a constant discount factor present value model for stock prices to construct and test variance bounds on stock prices. Because their approach is related to the one taken in this paper, a brief summary is worthwhile.

In the variance bounds tests, stock prices equal the expectation of the discounted present value of future cash flows (dividends):

$$P_t = E_t \left(\sum_{j=1}^{\infty} \beta^j D_{t+j} \right), \qquad (1)$$

where P_t is the real stock price, D_t is the real dividend, and β is the constant discount factor. E_t is a conditional expectation operator. Define the ex post series $P_t^* \equiv \sum_{j=1}^{\infty} \beta^j D_{t+j}$, and the following relationships result:

$$P_t^* = P_t + \eta_t \tag{2}$$

$$Var(P_t^*) = Var(P_t) + Var(\eta_t). \tag{3}$$

η_t is a forecast error which must be uncorrelated with the stock price, P_t, according to the rational expectations or market efficiency assumption. This model for stock prices can be viewed as the market fundamental with no arbitrary solution or speculative bubbles. The model in (1) is actually derived from a fundamental difference equation:

$$P_t = \beta E_t(P_{t+1} + D_{t+1}), \tag{4}$$

which is consistent with optimizing behavior of risk-neutral agents. Unfortunately, the price solution in (1) is only one of infinitely many solutions to the difference equation (4), but it is the common solution associated with market fundamentals. The following equation for stock prices also satisfies the difference equation:

$$P_t = E_t\left(\sum_{j=1}^{\infty} \beta^j D_{t+j}\right) + A_t,$$

for any A_t such that $E_t(A_{t+1}) = \frac{1}{\beta} A_t$. If the arbitrary solution is deterministic, it has the form $A_t = C(\frac{1}{\beta})^t$, but a deterministic bubble is implausible because consumer-investors would eventually cash in their assets and indulge themselves in a consumption binge. Blanchard and Watson describe a stochastic bubble which is explosive and satisfies the discounted martingale property, but eventually crashes with probability one. Indeed, a stochastic bubble could add enough variation to stock prices so that the variance relationship in (3) is violated.

Shiller and LeRoy and Porter find that estimated variances for stock prices violate the upper bound by dramatic margins. These tests have been the subject of considerable econometric criticism, but Scott (1985) has shown that the technical problems can be avoided by constructing a regression test of equation (2), and the results of these regression tests are equally dramatic.

Another interpretation of these empirical results which has gained wide acceptance emphasizes the role of risk aversion. LeRoy and LaCivita (1981) and Michener (1982) have shown that risk aversion can produce additional variation in stock prices so that the variance relationship in (3) is violated. In a world of risk aversion, the appropriate model of market fundamentals becomes

$$P_t = E_t\{ \sum_{j=1}^{\infty} \beta^j \frac{\lambda_{t+j}}{\lambda_t} D_{t+j} \}, \qquad (5)$$

where λ_t is related to the marginal utility of wealth, and the variability of stock prices depends on the variability of dividends and the marginal utility of wealth. The results of the variance bounds tests imply one or more of the following possibilities: (1) risk aversion with significant variation in marginal utility of wealth and real interest rates, (2) speculative bubbles, or (3) rejection of rational expectations and market efficiency.

In this paper, we focus on the bubbles hypothesis by incorporating risk aversion and varying real interest rates in the empirical test. The rational expectations assumption is required to estimate the parameters of an intertemporal utility function and a specification test for this part of the model is available. Given the parameters of the utility function, we model the marginal utility of wealth

variable and develop a regression test to determine whether stock prices contain a significant arbitrary solution (or stochastic bubble term). In essence, we employ an asset pricing model which is a variation on the consumption-based capital asset pricing model. In Section I, we develop the asset pricing model and discuss some of the technical issues associated with bubbles. In addition, we define two distinct interpretations for market efficiency or rational expectations in the stock market. In Section II, we describe the method of moments estimator and the econometric methods which are used to estimate the parameters of the utility function and to test for the significance of bubbles. In Section III, we present the results.

I. Intertemporal Asset Pricing and Speculative Bubbles in Rational Markets

Most asset pricing models revolve around a relationship of the following form:

$$\lambda_t P_t = \beta E_t[\lambda_{t+1}(P_{t+1} + D_{t+1})]. \tag{6}$$

This relationship can be deduced from models in which agents either maximize utility of terminal wealth or solve a consumption-investment problem in which utility of lifetime consumption is maximized. The relationship is a first order stochastic difference equation and a boundary condition is required in order to have a unique solution for prices. The solution in equation (5) is only one of many possible solutions, but it is the one associated with market fundamentals and from here on we shall define the right-hand side of (5) as the market fundamental. The general solution has the form:

$$P_t = E_t\left(\sum_{j=1}^{\infty} \beta^j \frac{\lambda_{t+j}}{\lambda_t} D_{t+j}\right) + \frac{1}{\lambda_t} A_t,$$

where A_t is the arbitrary solution and must satisfy the discounted martingale property $E_t(A_{t+1}) = \frac{1}{\beta} A_t$. The arbitrary solution also represents a speculative bubble, or a deviation from market fundamentals. If we begin with an ad hoc model in which agents maximize utility of next-period wealth and repeat this period after period, there is no terminal condition for ruling out speculative bubbles. In the Lucas model, however, we have agents maximizing over an infinite horizon and the transversality condition for the optimization problem, $\lim_{T\to\infty} \beta^T E_t(\lambda_{t+T} P_{t+T}) = 0$, forces A_t to be zero for all periods. Hence, the existence of a terminal or transversality condition rules out speculative bubbles and stock prices are based on market fundamentals only. This case corresponds to Tirole's (1982) fully dynamic rational expectations equilibrium in which bubbles do not exist.

This observation suggests that for models in which agents receive utility from terminal wealth, stock prices may contain speculative bubbles. In terms of real individuals and real investors, utility from terminal wealth corresponds to individuals deriving satisfaction from passing wealth on to their heirs. In the finance literature, Merton (1971, 1973) and others include a bequest function to account for utility of terminal wealth. To motivate the empirical tests in this paper, we have a simple asset pricing model based on the behavior of a representative agent. Imagine that we have a large number of agents who live T periods. At the end of T periods, these agents are

replaced by heirs and the dying individuals pass wealth on to their heirs. The representative agent for this economy solves the following consumption-investment problem:

$$\max_{C_t, \underline{a}_t} E_t\{\sum_{j=0}^{T} \beta^j U(C_{t+j},\cdot) + \beta^T V(W_{t+T})\},$$

subject to a budget constraint. C_t is consumption, $\underline{a}_t$ is a vector of shares representing shares held of all securities available, and $V(W_{t+T})$ is utility of terminal wealth.[5] At this stage of the analysis, we do not completely specify the utility function $U(C_{t+j},\cdot)$, other than to assume that the utility function is concave. We leave open the possibility that utility of consumption this period may depend on the level for the previous period so that C_{t+j-1} may be included in $U(C_{t+j},\cdot)$. The representative agent takes asset prices as given exogenously, but the collective behavior of all agents determines equilibrium prices. By studying the consumption-investment optimization problem of the representative agent, we can derive a relationship for stock prices. The first order conditions associated with optimal share allocations in each security have a form similar to that of (6):

$$\lambda_t P_{it} = \beta E_t[\lambda_{t+1}(P_{i,t+1} + D_{i,t+1})], \quad i=1,\ldots,N. \qquad (7)$$

λ_t is a LaGrange multiplier associated with the budget constraint in period t and is related to the marginal utility of wealth. In a rational expectations equilibrium, stock prices, consumption, and the marginal utility of wealth variable λ_t are determined so that this relationship is satisfied. By specifying the form of the utility

function, we can say more about λ_t. If utility of consumption is separable over time, for example, then λ_t equals $U'(C_t)$.

If the bequest function $V(W_{t+T})$ is taken to be the derived utility of initial wealth for our identical heirs, generation after generation, then the problem becomes an infinite-horizon problem which, as we have already noted, has a transversality condition which rules out arbitrary solutions. If agents get no utility from bequested wealth, then we have a terminal condition which rules out arbitrary solutions. The first order conditions associated with shares passed on to heirs have the following form:

$$E_t\{\beta^T P_{i,t+T}[\frac{\partial V}{\partial W_{t+T}} - \lambda_{t+T}]\} = 0,$$

where λ_{t+T} is the LaGrange multiplier for the budget constraint in period t+T and will equal marginal utility of consumption in the last period. If V = 0, then we have a terminal condition $E_t[\beta^T \lambda_{t+T} P_{i,t+T}] = 0$ which can be used to eliminate arbitrary solutions. If V depends on terminal wealth, this condition simply says that we expect to allocate wealth optimally in the last period between consumption and bequests and that marginal utility of consumption will equal marginal utility of bequested wealth; speculative bubbles are not ruled out. Agents in this model do not consider the ramifications of passing a bubble on to subsequent generations, and the presence or possibility of speculative bubbles suggests that agents are myopic. Alternatively, one could posit a representative-agent infinite-horizon model in which myopic agents ignore the transversality condition.

To develop an empirical test, we need a model for the marginal utility of wealth variable. A popular model in the literature has been a time-additive separable utility function for which $U(C_t)$ is a power function: $U(C_t) = \frac{1}{1-\alpha} C_t^{1-\alpha}$ for $\alpha \neq 1$ and $U(C_t) = \ell n\ C_t$ for $\alpha = 1$.[6] The recent empirical work on these models suggests that a separable utility function does not adequately fit the data on asset returns. Dunn and Singleton (1983, 1984) have recently examined asset pricing models in which utility of consumption is not separable. Indeed, many scholars have argued that the time-additive separable utility function is not realistic; Hicks (1965, pp. 261-62) argues that utility of consumption for a given period should depend on consumption for adjacent periods. Individuals have strong desires to maintain smooth, stable consumption spending plans and one indication of this behavior is the smooth pattern of aggregate consumption expenditures.[7] To capture this aspect of behavior, we use a utility function of the following form:

$$U(C_t,\cdot) = \frac{1}{1-\alpha}\left[C_t \exp\left\{-\alpha(\sqrt{\ell n\ C_t} - \sqrt{\ell n\ C_{t-1}})^2\right\}\right]^{1-\alpha}$$

$$= U(C_t, C_{t-1}, \alpha, \gamma), \qquad (8)$$

where $\alpha \geq 0$ and $\gamma \geq 0$. In the steady state, this utility function collapses to the familiar power utility function. The γ parameter measures the decrease in utility associated with large changes in consumption. If we take the logarithm of the argument for the power function, we have, $\ell n\ C_t - \gamma(\sqrt{\ell n\ C_t} - \sqrt{\ell n\ C_{t-1}})^2$. The second part of this expression is similar to an adjustment cost. We use the square root

form in order to avoid quadratic adjustment costs, which would imply that marginal utility of consumption eventually becomes negative as consumption increases. For the specification that we use, marginal utility of consumption remains positive at least for small values of γ.

When we use the utility function (8) in the dynamic optimization problem for our representative agent, we have the following first order condition associated with current consumption:

$$E_t\,(MU_t) = \lambda_t, \text{ where}$$

$$MU_t \equiv C_t^{-\alpha} \exp[-\gamma(1-\alpha)(\sqrt{\ell n\ C_t} - \sqrt{\ell n\ C_{t-1}})^2](1 - \gamma + \gamma\sqrt{\frac{\ell n\ C_{t-1}}{\ell n\ C_t}})$$

$$+ \beta\ C_{t+1}^{-\alpha} \exp[-\gamma(1-\lambda)(\sqrt{\ell n\ C_{t+1}} - \sqrt{\ell n\ C_t})^2]$$

$$\gamma\ \frac{C_{t+1}}{C_t}\left[\sqrt{\frac{\ell n\ C_{t+1}}{\ell n\ C_t}} - 1\right]$$

Using this result, we now have the following empirical relationship for asset prices:

$$P_{it}E_t(MU_t) = \beta E_t[MU_{t+1}(P_{i,t+1} + D_{i,t+1})],$$

or in terms of returns ($R_{i,t+1} \equiv \frac{P_{i,t+1} + D_{i,t+1}}{P_{it}}$):

$$E_t\{MU_t - \beta MU_{t+1}R_{i,t+1}\} = 0.$$

With this relationship, we can follow Hansen and Singleton (1982) and apply Hansen's (1982) generalized method of moments estimator to estimate the three parameters (β,α,γ).

One final point is necessary before turning to the empirical tests. There are several definitions of market efficiency in the

finance literature, and despite some occasional rejections, the empirical tests generally support the efficient markets hypothesis. To understand the importance of speculative bubbles, we find it useful to distinguish between two definitions for market efficiency. Definition 1 is due to Fama (1970) and states that excess market values, $P_{j,t+1} - E_t(P_{j,t+1})$, or excess returns, $R_{j,t+1} - E_t(R_{j,t+1})$, are fair games. In other words, the expectation of either one of these variables is zero given information available to investors at time t. The numerous tests of market efficiency are directed at this definition. In terms of our asset pricing model, the relationship in equation (7) must be satisfied. Sharpe, as well as others, has expanded the definition of market efficiency as follows:

> A (perfectly) efficient market is one in which every security's price equals its investment value at all times.[8]

This definition is a tautology unless we interpret investment value as our market fundamentals for stock prices. For definition 2, we say that stock prices equal the value determined by market fundamentals and that there are no speculative bubbles. To some readers, these two definitions may seem to be equivalent, but in fact the second definition is a much stronger statement. A stock market in which there are speculative bubbles which satisfy the discounted martingale property outlined above will be efficient according to definition 1, but not according to definition 2. The variance bounds tests of Shiller and LeRoy and Porter and the regression tests in Scott are directed at this second definition, with the added requirement that marginal utility of wealth is constant. For the sake of discussion, we shall refer to definition

1 as short-run market efficiency and definition 2 as long-run market efficiency.

II. The Empirical Tests

The empirical test that we develop for speculative bubbles is based on an ordinary least squares (OLS) regression in which our measure of ex post or realized market fundamentals is regressed on stock prices. First, we present the regression test for the case of a separable intertemporal utility function, and then we present the test for the more complicated non-separable utility function described in the previous section. For the separable utility function, we use the power (or constant relative risk aversion) utility function and $\lambda_t = C_t^{-\alpha}$ so that our model of market fundamentals is:

$$P_t = E_t\{\sum_{j=1}^{\infty} \beta^j (\frac{C_{t+j}}{C_t})^{-\alpha} D_{t+j}\}.$$

This is the model that Grossman and Shiller (1981) examine. Let $P_t' \equiv \sum_{j=1}^{\infty} \beta^j (\frac{C_{t+j}}{C_t})^{-\alpha} D_{t+j}$, the ex post market fundamental in this model. If stock prices are determined by market fundamentals only, then we have the following simple regression relationship:

$$P_t' = P_t + \eta_t,$$

where η_t is the forecast error associated with P_t'. If we run the following regression, the constant should equal zero and the slope coefficient should equal one: $P_t' = a + bP_t + \eta_t$. Because the forecast error should be uncorrelated with P_t, we have a regression equation that can be estimated by OLS. The error term, however, is serially correlated, but as we have noted elsewhere any attempt to

filter the equation to remove the serial correlation results in inconsistent parameter estimates.[9] As a result, we need to apply OLS and account for the effect of serial correlation in the variance matrix for the parameter estimates. If stock prices contain a significant bubble term, then P_t no longer equals $E_t(P_t')$ and the parameter estimates for a and b will be biased away from zero and one, respectively. Thus a test of $a = 0$ and $b = 1$ constitutes a test for the presence of a significant bubble term. Another interpretation of this regression is whether stock prices are an unbiased predictor of the future ex post market fundamentals. The problem with this approach is that we need an estimate of α, as well as β. Grossman and Shiller use values of 0, 1, and 4 for α and corresponding estimates of β, and we have followed this approach so that our tests can be compared with their analysis.[10]

A corresponding test with non-separable utility is slightly more complicated because $\lambda_t = E_t(MU_t)$ where MU_t depends on C_t, C_{t+1}, and C_{t-1}. To have an exact measurement of λ_t in this model specification, we would need to model the process generating the endogenous consumption variable. One of the attractive features of our empirical test is that we are not required to model the processes generating consumption, dividends, and the suspected stochastic bubble. In order to retain this feature of the test, we must add one more modification. In the previous section, we have shown that stock prices must satisfy the following relationship:

$$P_{it}E_t(MU_t) = \beta E_t[MU_t(P_{i,t+1} + D_{i,t+1})]. \qquad (9)$$

The solution to this stochastic difference equation has the following form:

$$P_{it}E_t(MU_t) = E_t\{\sum_{j=1}^{\infty} \beta^j MU_{t+j}D_{i,t+j}\} + A_{it}, \qquad (10)$$

where again the arbitrary solution must satisfy the discounted martingale property. If stock prices are characterized by the absence of bubble terms, then:

$$E_t\{\sum_{j=1}^{\infty} \beta^j MU_{t+j}D_{i,t+j} - P_{it}MU_t\} = -A_{it} = 0.$$

Again we create a series for the ex post or realized market fundamentals, $P^*_{it} = \sum_{j=1}^{\infty} \beta^j MU_{t+j}D_{i,t+j}$, but now we consider the data series $P^*_{it} - P_{it}MU_t$ by using the price solution in (10):

$$P^*_{it} - P_{it}MU_t = P^*_{it} - E_t(P^*_{it}) - A_{it} = \eta_{it} - A_{it}.$$

η_t is the forecast error for P^*_{it} and should be uncorrelated with information dated time t or earlier by the rational expectations or market efficiency principle. A regression of $P^*_{it} - P_{it}MU_t$ on variables dated time t or earlier should produce a zero intercept and zero slope coefficients if there is no bubble term. The bubble term, however, can be correlated with variables dated at time t, and if there is a bubble term, the proposed regression could produce significant coefficients if we include variables which are highly correlated with the bubble. A natural candidate would be the price variable itself, so that we have the following regression:

$$P^*_{it} - P_{it}MU_t = a + bP_{it} + e_t.$$

Again the error term e_t should be serially correlated because it will contain the forecast error η_t. The regression test with the separable CRRA utility function can be recast in this form by simply regressing

$P'_{it} - P_{it}$ on P_{it} and an intercept, and now the slope coefficient should be zero.

Kleidon (1982, 1985) and Marsh and Merton (1983, 1984) have criticized the variance bounds tests of stock prices by arguing that the price and dividend series are not stationary, even after the removal of a time trend. They argue that a more plausible approach is to model growth rates (or percentage changes) in dividends and stock prices as stationary time series, and they present evidence that their models are not rejected in favor of Shiller's time trend model. The parameter estimates in the regression tests that we propose are consistent if prices and dividends are nonstationary series, but it will be difficult to derive a non-degenerate asymptotic distribution for the estimates if the data series are not stationary. For the estimation in this paper, we adopt the assumption that growth rates in dividends, stock prices, and consumption are stationary. It follows that rates of return will be stationary. In addition, the price-dividend ratio must be stationary if there are no bubbles in stock prices. For the separable CRRA utility function, we divide by dividends to get the following regression:

$$\frac{P'_{it}}{\bar{D}_{it}} = a + b\,\frac{P_{it}}{\bar{D}_{it}} + e_t.$$

For the non-separable utility function, we make two adjustments on the data. First we take the dependent variable $P^*_{it} - P_{it}MU_t$ and divide by $UC_t = C_t^{-\alpha}\exp[-\gamma(1-\alpha)(\sqrt{\ln C_t} - \sqrt{\ln C_{t-1}})^2](1 - \gamma + \gamma\sqrt{\frac{\ln C_{t-1}}{\ln C_t}})$, which is the first term in MU_t and does not depend on future consumption, C_{t+1}. Then we divide both the dependent variable and stock prices by dividends to get:

$$\frac{P^*_{it} - P_{it}MU_t}{UC_t \bar{D}_{it}} = a + b\frac{P_{it}}{\bar{D}_{it}} + e_t$$

The dividend series that we use for $\bar{D}_{it}$ to account for a heteroskedasticity is dividends cummulated over four quarters ($\bar{D}_{it} = D_{it} + D_{i,t-1} + D_{i,t-2} + D_{i,t-3}$). If we were to use quarterly dividends ($\bar{D}_{it} = D_{it}$), we would be adding some additional seasonality to the data. Because this procedure cannot be applied to companies that have not paid regular dividends, we have restricted our sample to companies that pay regular dividends. One alternative would be to divide by some measure of earnings.

One final problem concerns the calculation of the series P^*_{it} and P'_{it} from finite data series. We follow Grossman and Shiller and use the stock price at the end of the sample period to estimate P^*_{it} and P'_{it} beyond the sample period. For P^*_{it}, we have:

$$\hat{P}^*_{i,T+1} = MU_{T+1}P_{i,T+1}$$

$$\hat{P}^*_{iT} = \beta\hat{P}^*_{i,T+1} + \beta MU_{T+1}D_{i,T+1}$$

$$\hat{P}^*_{it} = \beta\hat{P}^*_{i,t+1} + \beta MU_{t+1}D_{i,t+1}$$

$$\vdots$$

$$\hat{P}^*_{i1} = \beta\hat{P}^*_{i2} + \beta MU_2 D_{i2}$$

to generate $\hat{P}^*_{it}$, $t = 1,\ldots,T$. A similar procedure is used for $\hat{P}'_{it}$. The same results that we have for P^*_{it} also apply to our estimate $\hat{P}^*_{it}$: namely $E_t(P^*_{it} - P_{it}MU_t) = 0$ and $E_t[\frac{\hat{P}^*_{it} - P_{it}MU_t}{UC_t\bar{D}_{it}}] = 0$ if there are no

bubbles. As a result, the regression relationship still holds with $\hat{P}^*_{it}$ in place of $\hat{P}^*_{it}$.[11]

Up to this point, we have assumed knowledge of the parameter values (β, α, γ), but in fact we need to estimate these parameter values. Here we must use the concept of short market efficiency and the asset pricing relation: $E_t\{MU_t - \beta MU_{t+1} R_{i,t+1}\} = 0$. Again we divide by UC_t and form the following variable which depends on (β,α,γ):

$$U_t = \frac{MU_t}{UC_t} - \beta \frac{MU_{t+1}}{UC_t} R_{i,t+1}.$$

Since $E_t(U_t) = 0$, we can apply the instrumental variables procedure of Hansen and Singleton to estimate (β, α, γ). For the non-separable utility function, U_t is a first order moving average process and we find that we must use the spectral estimator to compute the optimal weighting matrix for the GMM estimator.[12] We also iterate on the optimal weighting matrix and the parameter estimates converge in less than ten iterations. For instruments, we use a constant, $(R_{it}-1)$, $(\frac{C_t}{C_{t-1}} - 1)$, and RF_t, a short-term interest rate of Treasury bills. Several studies have found significant correlations between stock returns and short-term interest rates known at the beginning of the period.

Our approach to testing for bubbles is thus a two-stage approach. First, we estimate the parameters of the utility function with the GMM estimator. Then taking these values as given, we run the simple regression to test for the presence of bubbles. The estimation error for the utility function parameters does have an effect on the

variance matrix for the OLS regression estimates. By observing that both the first and second stage estimators are GMM estimators, we can consider a joint GMM estimator. It can be shown that the separate optimal GMM estimator for (β,α,γ) is also the optimal joint GMM estimator, and the variance matrix for $(\hat{\beta},\hat{\alpha},\hat{\gamma})$ is the same. One can see this intuitively by noting that in the regression equation, there are no orthogonality conditions which will permit us to identify any of the utility function parameters. The separate OLS estimates for the regression equation are not the optimal joint GMM estimator, but the separate OLS estimator is much easier to calculate and it is possible to construct the appropriate variance matrix within the context of the joint GMM estimator. The details for the joint GMM estimators and the variance matrices are shown in an appendix.

III. The Results

The data for our empirical tests of bubbles consist of quarterly observations for the period 1947-83 on the following variables: prices and dividends for a NYSE portfolio and twenty individual companies, consumption expenditures and deflators for nondurables and services, population, and the interest rate on short-term Treasury bills. The price and dividend series for the NYSE are computed by using the returns, with and without dividends, on the value-weighted portfolio of all stocks on the NYSE found on the CRSP monthly file.[13] Prices and dividends for individual companies are from the CRSP monthly tape, with prices and dividends adjusted for stock splits and stock dividends. Real consumption expenditures, consumption deflators, and population are from the Citibank database. Real consumption

on nondurables plus services is used for the consumption variable and the corresponding deflator is used to convert nominal prices, dividends, and returns into real quantities. The population series is used to compute per capita real consumption. Our use of per capita real consumption on nondurables plus services as the consumption variable involves an implicit assumption that utility of consumption is separable across durables and nondurables plus services. The interest rate on one-month T-bills (for the beginning of each quarter is taken from the study by Ibbotson and Sinquefield and supplemented with rates from the Wall Street Journal.

First, we present the results for utility modeled as a separable CRRA utility funcion, with values of 0, 1, and 4 for the RRA parameter. The value of zero corresponds to risk neutrality and the constant discount factor model. The results are shown in Table 1. The estimate of β is computed as follows:

$$\hat{\beta} = \frac{1}{\frac{1}{T}\sum_{t=1}^{T}\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha} R_{t+1}}.$$

This estimate is a GMM estimator and we account for the estimation variance of $\hat{\beta}$ in the variances for the estimates of a and b in the OLS regression. The t tests for a = 0 and b = 1 are all significant at standard significance levels: the t statistics for b = 1 range from -9.76 to -7.24. These results imply rejection of the null hypothesis of no bubbles within the separable CRRA model. A χ^2 statistic for the joint test of a = 0 and b = 1 has been computed and is included in the table. We have also included the sample variances for $P'_t/\bar{D}_t$ and $P_t/\bar{D}_t$

so that one can compare the regression results with the results of a variance bounds test. Here the estimated variance of the price-dividend ratio exceeds the estimate of its upper bound by a large factor. As we increase the RRA parameter, we increase the estimated variance of the upper bound and there may be some large value for the RRA parameter which rationalizes the variability of the price-dividend ratio. These results are consistent with the graph in Grossman and Shiller for the period after World War II. It is worth noting that we no longer get a neat variance bound relationship when the utility function is not separable.

In Table II, we present the results for the NYSE portfolio. The estimates for (β, α, γ) are at the top of the table. α and γ are not estimated with much precision, but the γ estimate is 2.53 standard errors away from zero suggesting that this consumption smoothing parameter is statistically significant. The χ^2 specification test statistic is low and does not indicate rejection of this part of the model. The GMM estimator sets the parameters so that sample average of U_t and the covariances of U_t with $(R_t - 1)$, $(C_t/C_{t-1} - 1)$, and RF_t are all close to zero. The χ^2 test is effectively a measure of whether these sample moments are significantly different from zero. As an additional check, we have computed the covariances of U_t with $(R_{t-1} - 1)$, $(C_{t-1}/C_{t-2} - 1)$, and RF_{t-1}, which are not used in setting the parameter estimates. The correlation coefficients of U_t with $(R_t - 1)$, $(C_t/C_{t-1} - 1)$, and RF_t are -.036, -.025, and .099, respectively, and the correlation coefficients of U_t with $(R_{t-1} - 1)$, $(C_t/C_{t-1} - 1)$, and RF_{t-1} are .062, .041, and .094, respectively. The inclusion of addition lags for the instrumental variables will not alter the outcome of

the specification test. Hansen, Singleton, and Dunn find that the specification tests indicate rejection for many of the models that they estimate. In the regression test for the presence of bubbles, we estimate an intercept of 21.307 and a coefficient of -1.2449 for the price-dividend variable. The t statistics for both coefficients indicate rejection of the null hypothesis of no bubbles. Here we use the concept of short-run market efficiency to estimate the utility function parameters, but when we use these estimates and the same data, we find dramatic rejection of the concept of long-run market efficiency in which stock prices are determined by market fundamentals only. In addition to dividend variability, variability in marginal utility of wealth determines the variability of market fundamentals for stock prices. For a check on the data, we have compared our MU_t series with C_t^{-1} and C_t^{-4}, two measures for marginal utility of wealth in the separable CRRA model. We calculate the following variances for growth rates: .004744 for MU_t, .0003488 for C_t^{-1}, and .0005496 for C_t^{-4}. Although the variability of MU_t is much greater than that for the CRRA model, our marginal utility of wealth variable cannot explain the variation of stock prices.

The test using the NYSE data examines the existence of a bubble that is common to the stocks of the NYSE. In Table III, we examine tests for individual bubbles. At the first stage of estimating the utility function parameters, we encounter some minor computational problems. Conceivably, one can construct a U_{it} series for all 20 stocks plus the NYSE and use a corresponding set of instruments to set up a large number of orthogonality conditions. This procedure has

been attempted, but we are not able to invert the optimal weighting matrix when we use more than two securities (roughly eight orthogonality conditions). To compromise, we have used the returns on the NYSE portfolio and returns on an equally-weighted portfolio of the 20 companies used in the sample. Let U_{1t} be the error term for the NYSE and U_{2t} be the error term for the equally-weighted portfolio. The instruments for U_{1t} include a constant, $(R_{1t} - 1)$, $(C_t/C_{t-1} - 1)$, and RF_t, and the instruments for U_{2t} are the same except that R_{2t} replaces R_{1t}. The estimates of β, α, γ are presented at the top of the table. The estimate for α changes from 1.3449 to 2.1161. Again the χ^2 specification test indicates acceptance of this part of the model: short-run market efficiency is not rejected by the data. The OLS estimates for the regression equation for the NYSE and the 20 companies are included in the second portion of this table. Again, the standard errors and t statistics for a and b incorporate the estimation variance in $\hat{\beta}$, $\hat{\alpha}$, and $\hat{\gamma}$. The results for the NYSE are almost identical to those in Table II. The results for the 20 companies are just as dramatic as those for the NYSE. Out of the 20, there are only four stocks (Exxon, General Motors, IBM, and U.S. Tobacco) for which the null hypothesis of no bubbles is accepted. The results for the other sixteen companies indicate evidence that stock prices do contain speculative bubbles. Fifteen of the coefficients on the price-dividend ratios are significant at the 1% level.

For a final comparison of stock prices with ex post market fundamentals, we present a plot of real stock prices, P_t, versus a rough estimate of the ex post market fundamentals $\hat{P}^*_t/\lambda_t$. As we noted in the

previous section, we do not have an exact measure of λ_t; what we have is a series MU_t (which includes C_t, C_{t+1}, and C_{t-1}) such that $E_t(MU_t) = \lambda_t$. If we ignore the forecasting problem with C_{t+1}, we can use MU_t as a crude measure for λ_t and construct a series $\frac{P_t^*}{MU_t}$. If market fundamentals only are important, then real stock prices should equal the conditional expectation of P_t^*/λ_t and should track with our crude estimate $\hat{P}_t^*/MU_t$. In Figure 1, we present a plot of P_t with $\hat{P}_t^*/MU_t$ for the NYSE portfolio and the estimates in Table II. In Figure 2, we present a plot of these series deflated by dividends, $\frac{\hat{P}_t^*}{MU_t \bar{D}_t}$ and $\frac{P_t}{\bar{D}_t}$. From the graphs, one can see that stock prices are reasonably close to ex post market fundamentals if we exclude the twenty-year period 1954-73. During this twenty-year period, stock prices appear to be much too high relative to underlying market fundamentals. This same twenty-year bubble is also identified in the work of Shiller and Grossman and Shiller.

IV. Conclusions

The results presented in Section III indicate strong evidence that stock prices have deviated substantially from underlying market fundamentals during the post World War II period in the United States. Throughout the paper, we have interpreted this pathological behavior as evidence of speculative bubbles even though we have not provided a direct test for bubbles. We adopt this interpretation because stock prices over the same period satisfy our concept of short-run market efficiency, but do not satisfy our concept of long-run market efficiency. As we have argued, the existence of speculative bubbles which

satisfy the discounted martingale property would result in acceptance of short-run market efficiency and rejection of long-run market efficiency. We need the concept of short-run market efficiency or rational expectations in order to identify and estimate the parameters of the intertemporal utility function. The tests, of course, depend on the validity of the model used, particularly the non-separable utility function that we use to measure marginal utility of wealth, but we find that a specification test does not reject this part of the model. We have noted that our measure of marginal utility of wealth is much more variable than the corresponding measure for a separable CRRA utility function, but this additional variation cannot explain the variation of stock prices. We believe that it will be virtually impossible to construct a plausible marginal utility of wealth variable that will explain the behavior of stock prices over this period.

We leave to further research the study of the ramifications of speculative bubbles in financial markets, but we do note that these results indicate an important role for fundamental security analysis. One final point needs to be emphasized: the results of this paper do not refute the results of the market efficiency studies in finance which have generally supported the concept of short-run market efficiency. Once we consider the possibility of speculative bubbles, it is clear why our results and those of the variance bounds tests are so dramatically different from those in the efficient markets literature. The differences are due, not to the power of any particular test statistic, but rather to different concepts of market efficiency. Our regression tests and the variance bounds tests examine a much stronger

statement about the behavior of stock prices. There are numerous episodes of speculative price explosions in economic history, and unfortunately it appears that modern financial markets are no more immune than were the markets of our ancestors.

FOOTNOTES

[1]For a discussion of these episodes, see MacKay (1932) and Kindleberger (1978).

[2]See the papers by Azariadis (1981), Blanchard (1979), Kass and Shell (1983), Harrison and Kreps (1978), and Triole (1982, 1985). Some writers have referred to the phenomena of speculative bubbles as extrinsic uncertainty or sunspots.

[3]This point is also made in Proposition 3 of Tirole (1982, pp. 1172-73).

[4]More recently, West (1985) has constructed a specification test for bubbles using the constant discount factor model and a linearized approximation for the model with varying real interest rates. By contrast, we directly attack the varying real interest rate problem by developing the test within a multi-period asset pricing model. The approach of West also requires an explicit model of the dividend process.

[5]An alternative story might be that the economy has a small portion of individuals who die each period and are replaced by heirs. Let T be the average remaining life and assume that the representative agent for the economy faces the optimization problem above each period.

[6]See Hansen and Singleton (1982).

[7]More recently, Bergman (1985) has examined that effects of non-separable utility on asset pricing models.

[8]Sharpe (1985, p. 67).

[9]See Scott (1985).

[10]We have attempted to estimate α and β jointly using our sample data, but we get negative estimates for α and β estimates greater than one. Other researchers have encountered similar problems with estimates of the separable CRRA model. See the tables in the errata (Econometrica, January 1984) for the work of Hansen and Singleton.

[11]Flood and Hodrick (1985) have noted that empirical constructions of ex post market fundaments like $\hat{P}_t$ contain the bubble, A_{T+1}, at the end of the sample under the alternative hypothesis of rational bubbles. Their analysis shows that rejection of the null hypothesis in the regression test also implies rejection of rational bubbles. This is not a problem if the bubble has collapsed and A_{T+1} is close to zero for our sample. In addition, we have added simulated rational bubbles to the model used in the Monte Carlo study in Scott (1985), and we find

that the null hypothesis is frequently (23 out of 50 simulations) rejected in the regression equation when rational bubbles are included. The null hypothesis tends to be accepted when the simulated bubble does not collapse during the sample period.

[12]To estimate the spectral density matrices, we apply the method of prewhitening and recoloring described in Nerlove, Grether, and Carvalho (1979, pp. 67-68). Fourth-order autoregressions are used to prewhiten the series.

[13]To construct the price series, we use $P_t = 95.18$, the composite index level at the end of 1983. We then compute $P_{t-1} = P_t/(1+r_{2t})$, where r_{2t} is the rate of return on the CRSP index, excluding dividends. For dividends, we compute $D_t = P_{t-1}(r_{it} - r_{2t})$, where r_{1t} is the rate of return including dividends. The resulting price and dividend series are adjusted for stock splits and stock dividends.

TABLE I

Separable CRRA Utility Function
NYSE Portfolio 1947-83

$$\frac{\hat{P}'_t}{\bar{D}_t} = a + b\frac{P_t}{\bar{D}_t} + e_t$$

α	0	1	4
$\hat{\beta}$	.9807	.9851	.9982
$S(\hat{\beta})$	.006962	.006943	.007168
a	22.63	23.37	25.42
$S(\hat{a})$	6.604	6.794	7.846
b	-.1684	-.2085	-.3218
$S(\hat{b})$	.1197	.1349	.1826
σ_e^2	4.371	5.100	8.957
t(a=o)	3.43	3.44	3.24
t(b=1)	-9.76	-8.96	-7.24
$\chi^2(2)$	170.4	153.4	126.7
$Var(\hat{P}'_t/\bar{D}_t)$	5.568	6.957	13.424
$Var(P_t/\bar{D}_t)$	44.309	44.309	44.309
Without $Var(\hat{\beta})$			
$S(\hat{a})$	2.994	3.314	4.082
$S(\hat{b})$	.1081	.1197	.1525

Sample Size = 147

TABLE II

Nonseparable Utility Function
NYSE Portfolio 1947-83

GMM Estimates of Utility Function Parameters

$$U_t = \frac{MU_t}{UC_t} - \beta \frac{MU_{t+1}}{UC_t} R_{t+1}$$

Four Instruments: Constant, $(R_t - 1)$, $(\frac{C_t}{C_{t-1}} - 1)$, RF^*_t

$\hat{\beta} = .9818$ (.01546) $\quad \hat{\alpha} = 1.3449$ (3.7388) $\quad \hat{\gamma} = 30.224$ (11.944)

$\chi^2(1) = 2.27 \qquad T=145$

Regression Equation

$$\frac{P^*_t - MU_t P_t}{UC_t \bar{D}_t} = a + b \frac{P_t}{\bar{D}_t} + e_t$$

$\hat{a} = 21.307$ (8.735) $\quad \hat{b} = -1.2449$ (.3047)

$t(a) = 2.44 \qquad t(b) = -4.09$

$\chi^2(2) = 22.99 \qquad \sigma^2_e = 10.680$

Standard errors are in parentheses. The standard errors for $\hat{a}$ and $\hat{b}$, without allowing for the variance of $\hat{\beta}$, $\hat{\alpha}$, and $\hat{\gamma}$, are 7.999 and .2907, respectively.

*RF_t is the interest rate in effect at the end of quarter t for a one-month T-bill.

TABLE III

Nonseparable Utility Function
NYSE Portfolio and Twenty Common Stocks
1947-83

GMM Estimtes of Utility Function Parameters

NYSE Portfolio and Equally-Weighted Portfolio
of Common Stocks (8 Orthogonality Conditions)

$\hat{\beta}$ = .9891 (.01357) $\quad$ $\hat{\alpha}$ = 2.1161 (3.4468) $\quad$ $\hat{\gamma}$ = 27.382 (9.496)

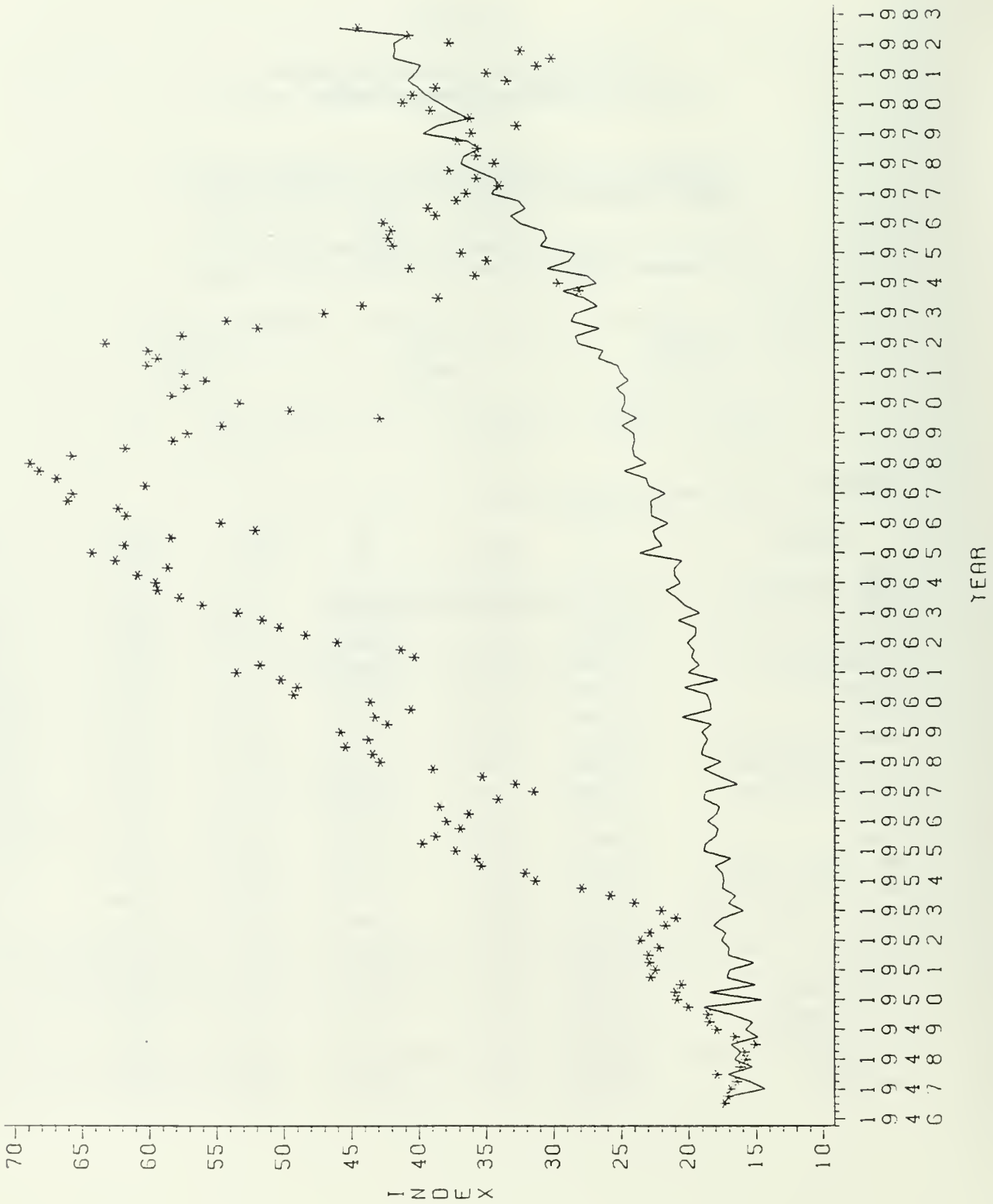
$\chi^2(5) = 5.14$ $\quad$ T = 145

Standard errors are in parentheses.

Stock	$\hat{a}$	$t(\hat{a})$	$\hat{b}$	$t(\hat{b})$	$\chi^2(2)$	σ_e^2
NYSE Composite	24.291	3.25	-1.2768	-5.16	28.48	8.617
AT&T	12.433	5.93	-1.0219	-6.22	50.17	1.423
Bethlehem Steel	17.824	2.27	-1.3742	-3.76	35.28	39.472
Columbia Gas	9.846	4.52	-.9058	-4.01	20.82	5.386
DuPont	22.683	4.76	-1.3513	-7.61	64.96	22.291
Exxon	4.349	.42	-.3464	-.69	.53	88.845
General Electric	32.018	3.38	-1.3775	-5.76	37.16	31.692
General Foods	22.944	1.99	-1.2267	-3.21	14.83	18.713
General Motors	7.927	.97	-.6149	-1.24	1.66	79.703
IBM	71.705	.77	-.7454	-.97	.96	4964.792
Kodak	33.542	2.29	-1.2290	-5.91	75.51	70.389
May Dept. Stores	18.226	1.55	-1.1282	-2.51	12.75	23.249
Owens Illinois	19.710	3.50	-1.1973	-6.37	48.50	7.401
Pacific Gas & Elec.	11.617	4.38	-.9893	-6.39	42.92	4.460
RCA	17.337	3.50	-.9755	-6.31	39.89	128.164
Texaco	17.391	1.97	-1.0404	-2.66	7.40	73.923
Union Camp	12.995	1.91	-.6790	-3.72	14.68	13.529
U.S. Tobacco	6.564	.35	.1836	.29	1.10	55.229
Westinghouse	24.208	2.34	-1.2615	-5.16	57.00	23.723
Woolworth	16.600	4.13	-1.2099	-6.54	45.55	5.549
Wrigley	14.401	1.81	-1.0969	-3.24	20.25	9.298

FIGURE 1

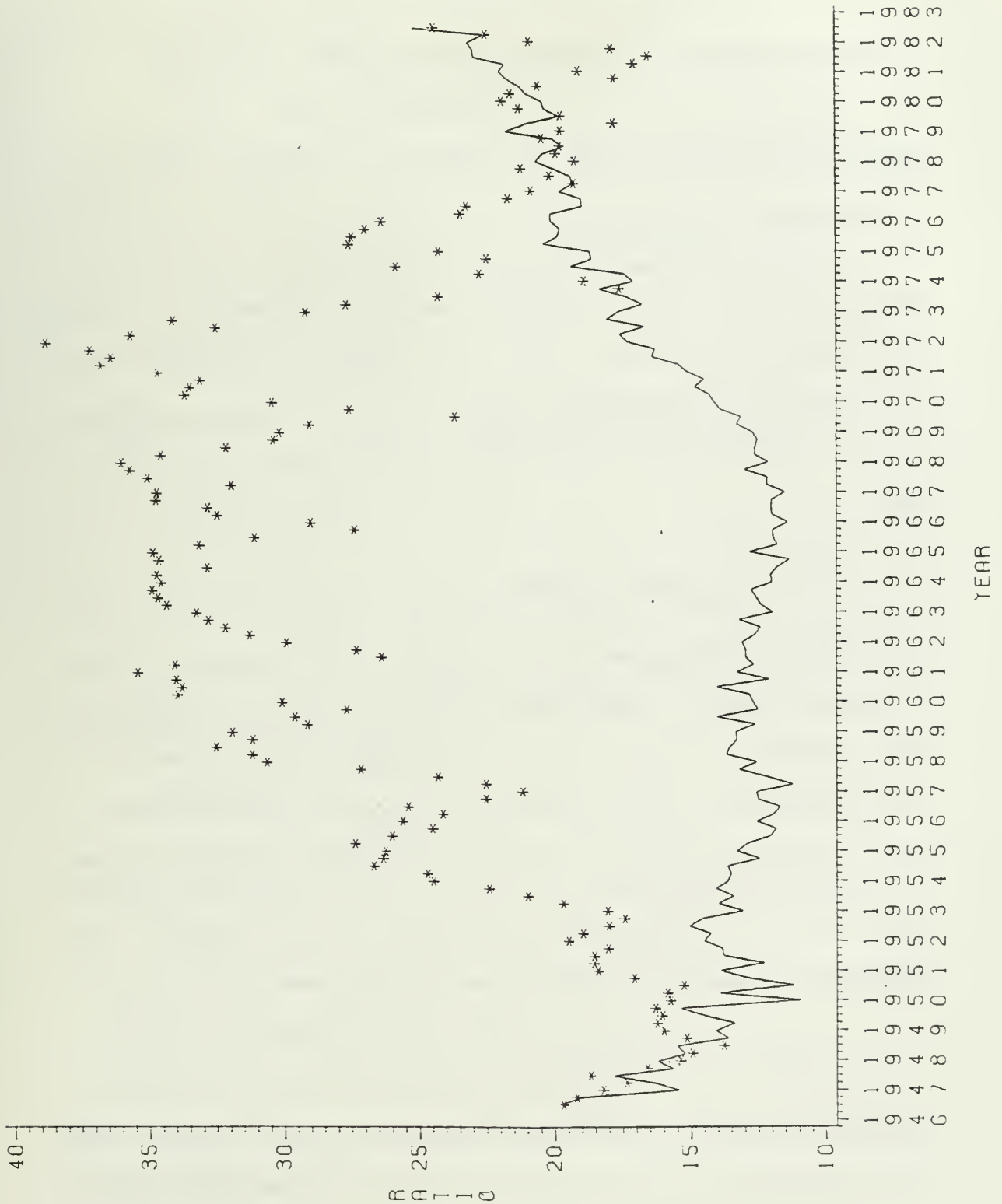
Stock Prices Versus Ex-Post Market Fundamentals, 1947-83



NOTE: The astericks are real stock prices (NYSE Composite), P_t. The solid line is a plot of the rough estimates of ex-post market fundamentals, $\hat{P}^*_{t/MU_t}$.

FIGURE 2

Price-Dividend Ratios Versus Ex-Post Market Fundamentals, 1947-83



NOTE: The astericks are the price-dividend ratios, $P_t/\bar{D}_t$. The solid line represents the rough estimates of the corresponding ex-post market fundamentals, $\frac{P^*_t}{MU_t\bar{D}_t}$.

APPENDIX

Here we present the details of the joint GMM estimator for $\underline{\theta}' = (\beta,\alpha,\gamma,a,b)$. Let $\underline{\theta}_1' = (\beta,\alpha,\gamma)$ and $\underline{\theta}_2' = (a,b)$. For $\underline{\theta}_1'$, we use the GMM-IV estimator of Hansen and Singleton (1982):

$$\min_{\underline{\theta}_1} \ell = \frac{1}{T} \underline{u}'ZW_{11}Z'\underline{u}$$

where $\underline{u}$ is a Tx1 vector containing u_t and Z is a TxK matrix containing the instrumental variables. The typical Z matrix for this paper has a t'th row of the following form: $\underline{z}_t' = (1,(R_t - 1), (C_t/C_{t-1}), RF_t)$. W_{11} is a weighting matrix, and for the optimal estimator $W_{11} = S_{11}^{-1}$ where

$$S_{11} = \sum_{j=-1}^{1} E(U_t U_{t-j} \underline{Z}_t \underline{Z}_{t-j}').$$

The spectral estimator makes use of the observation that S_{11} is also 2π times the spectral density matrix of $\underline{z}_t U_t$. For S_{11}, we use a sample estimate, and we iterate on the estimated weighting matrix. The GMM estimator of $\underline{\theta}_1$ is computed by using the Davidon-Fletcher-Powell routine to minimize ℓ; the algorithm uses analytical first partial derivative, $\partial\ell/\partial\underline{\theta}_1$.

The regression equations all have the following form:

$$y_t(\theta_1) = a + bx_t + e_t.$$

Let X be a Tx2 matrix containing a constant and x_t and let $\underline{e}$ be a vector containing e_t. The OLS estimator sets the orthogonality conditions $(\frac{1}{T} X'\underline{e})$ equal to zero. Let S_{22} equal 2π times the spectral

density matrix of $\begin{pmatrix} e_t \\ x_t e_t \end{pmatrix}$. If we take $\underline{\theta}_1$ as given, we have the following variance matrix.

$$\text{Var}\begin{pmatrix} a \\ b \end{pmatrix} = T(X'X)^{-1} S_{22} (X'X)^{-1}.$$

For our joint GMM estimator, we set $\underline{\theta}$ by minimizing all the orthogonality conditions simultaneously: $(\frac{1}{T} Z'\underline{u})$ and $(\frac{1}{T} X'\underline{e})$. Our approach to the GMM estimation of $\underline{\theta}$ employs the following weighting matrix:

$$W^{-1} = \begin{bmatrix} S_{11} & 0 \\ 0 & S_{22} \end{bmatrix}$$

The variance matrix for $[(\frac{1}{\sqrt{T}} Z'\underline{u}), (\frac{1}{\sqrt{T}} X'\underline{e})]$ has the following form:

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

where S is also 2π times the spectral density matrix of $\begin{bmatrix} \underline{z}_t & U_t \\ & e_t \\ x_t & e_t \end{bmatrix}$. An optimal estimator would S^{-1} for the weighting matrix, but we use the sub-optimal estimator with

$$W = \begin{bmatrix} S_{11}^{-1} & 0 \\ 0 & S_{22}^{-1} \end{bmatrix},$$

because it is easier to compute and has a nice intuitive interpretation. We doubt that the optimal joint GMM estimator for $\underline{\theta}$ would make much of an improvement. If we use only three instruments in Z, then our suboptimal estimator is the optimal estimator. We do need to calculate the variance matrix for the estimates of $\underline{\theta}$. From Hansen (1982), we have the following expression for the asymptotic variance matrix of $\sqrt{T}(\hat{\underline{\theta}}-\underline{\theta})$:

$$(d_0'Wd_0)^{-1}d_0'WSWd_0(d_0'Wd_0)^{-1}$$

where $d_{11} = \frac{1}{T} Z' \frac{\partial u}{\partial \underline{\theta}_1}$ $\qquad$ $d_{12} = \frac{1}{T} Z' \frac{\partial u}{\partial \underline{\theta}_2} = 0$

$d_{12} = \frac{1}{T} X' \frac{\partial e}{\partial \underline{\theta}_1}$ $\qquad$ $d_{22} = \frac{1}{T} X' \frac{\partial e}{\partial \underline{\theta}_2}$

After some rather tedious matrix algebra with partitioned matrices, we have the following expressions:

$$\text{Var}(\sqrt{T}(\hat{\underline{\theta}}_1 - \underline{\theta}_1)) = (d_{11}' S_{11}^{-1} d_{11})^{-1} \equiv G$$

$$\text{Var}(\sqrt{T}(\hat{\underline{\theta}}_2 - \underline{\theta}_2)) = d_{22}^{-1} S_{22} d_{22}^{-1} + d_{22}^{-1}[d_{21} Gd_{21}'$$

$$- S_{21} S_{11}^{-1} d_{11} Gd_{21}' - d_{21} Gd_{11}' S_{11}^{-1} S_{12}]d_{22}^{-1}$$

For hypothesis testing in large samples we have the following variance matrices:

$$\text{Var}(\hat{\underline{\theta}}_1) = T[(\frac{\partial u}{\partial \underline{\theta}_1})' Z S_{11}^{-1} Z' (\frac{\partial u}{\partial \underline{\theta}_1})]^{-1}$$

$$\text{Var}(\hat{\underline{\theta}}_2) = T\{(X'X)^{-1} S_{22}(X'X)^{-1} + (X'X)^{-1}[d_{21} Gd_{21}'$$

$$- S_{21} S_{11}^{-1} d_{11} Gd_{21}' - d_{21} Gd_{11}' S_{11}^{-1} S_{12}](X'X)^{-1}\}.$$

The expression for $\text{Var}(\hat{\underline{\theta}}_1)$ is the same expression for the optimal estimator which ignores the estimation of $\underline{\theta}_2$.

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